Money Talks*

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Abstract

We study the role of monetary policy in a model where aggregate risk can lead to a discrepancy between investors' beliefs and true fundamentals, and in which investment decisions are subject to a coordination problem. We formalize the notion that monetary policy is equivalent to information transmission by the central bank. To credibly communicate its information to investors, the Central Bank must apply monetary policy that violates the Friedman rule.

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1 Introduction

How can a Central Bank effectively communicate its own information about economic fundamentals to the private sector? What is the role of monetary policy in achieving efficient outcomes in the presence of aggregate risk and of potential coordination problems faced by investors in the economy? These questions are highly topical and have a long history in economics. However, decisive answers still eludes us. Part of the difficulty is in developing a tractable general equilibrium model that has the following three ingredients: (1) money plays an essential role in facilitating trade, (2) there is aggregate risk about fundamentals and information about this risk is dispersed, and (3) investment decisions are subject to a coordination problem, implying that individually optimal decisions might not maximize aggregate welfare. The goal of this paper is to provide such a model.[1]

We build an intertemporal model that is subject to frictions that make the use of money necessary in trade. Investors have heterogeneous expectations about economic fundamentals that affect investment returns, and they suffer a cost if their expectations are wrong. A benevolent Central Bank has the ability to print money and to provide loans to the private sector. It also has some information about the true state of the economy. We demonstrate that this information cannot always be credibly transmitted to the private sector through a simple announcement. This is because, under certain conditions, a benevolent Central Bank will have the incentive to misrepresent its confidence in the precision of its information. We demonstrate that credible information transmission by the Central Bank can be accomplished through a shift in its monetary policy. Thus, one message of this paper is that such information transmission must be exercised carefully as it creates a trade-off. Monetary policy conveys

[1] See Weiss (1980) and Barro and Gordon (1983) for two notable early attempts.

information that can enable more efficient investment and better coordination among investors. At the same time, it creates a distortion and may result in higher inflation in the economy. The study of optimal monetary policy concerns the search for the optimal way to balance the resulting benefits and costs.

Finally, we provide a formal criterion of the equivalence between monetary policy and information transmission by the Central Bank. This criterion requires that the set of feasible monetary policies is homeomorphic to the set describing the Central Bank's information. Whenever our criterion applies, at the cost of imposing a distortion associated with inflation, the Central Bank can use monetary policy in order to credibly transmit its information to the private sector.

The paper proceeds as follows. Section 2 describes the economic environment and Section 3 considers the full information benchmark. Section 4 studies the model under private information. Finally, Section 5 discusses our general equivalence result. A brief conclusion follows. The Appendix contains some of our proofs.

2 The Environment

Our basic model uses the setup developed by Berentsen and Monnet (2007), who in turn build on Lagos and Wright (2005) and on Kiyotaki and Wright (1989). While we believe that other choices of a monetary model are also consistent with our main findings, our choice was for the most tractable general equilibrium model where we can study information transmission and in which money is essential in trade.[2]

Time $t = 0, 1, ...,$ is infinite. The economy consists of a continuum of infinite-lived agents. There is a benevolent *Central Bank (CB)* which has the ability to print money. The CB lives for one period only and is replaced by a new CB at the end

[2] Indeed, our monetary model can be easily converted to a cash-in-advance economy.

of each period.[3] Each period, there are three stages: 0, 1, and 2. There are three goods: an *investment good*, k, a *stage-1-good*, q, and a *stage-2-good*, z. Investment takes place in stage 0 of each period. In stage 1, a market for good q opens. In stage 2, investment pays off in units of good z. Future periods are discounted at rate $\beta \in (0,1)$. There is no discounting between stages. We consider each stage in more detail next.

In stage 0 of each period half of the agents are randomly chosen to be producers (*investors*). Each investor i chooses how much of an investment good, k_i, to produce. The utility cost of producing k_i units of the investment good is $c(k_i)$, where $c(\cdot)$ is strictly increasing and strictly convex. All investments mature in stage 2. The return on this investment is uncertain and is given by θ^2 units of good z per unit of k, where θ is a random variable with an improper uniform prior on $(-\infty, +\infty)$. Nature draws θ at the start of stage 0.

The random variable θ is our way of introducing aggregate risk about the profitability of investment, an effect emphasized by Keynes (1936). Some (diverse) information regarding this risk is available in the economy. More precisely, when the true state is θ, the CB receives a signal $y = \theta + \eta$, while investors receive a signal $x_i = \theta + \varepsilon_i$. These signals are received in stage 0 prior to the investment decisions.[4] We assume that the noise terms η and ε_i are normally distributed with mean zero

[3] Alternatively, we can think of the CB's discount rate as being equal to zero. This allows us to completely abstract from reputation effects, which have been the focus of study in several other papers. In the case of a long-lived CB, truthful transmission of its information can be assured if (sufficiently patient) investors trigger a severe penalty if they discover that the CB miscommunicated its information in the past. Our analysis focuses on achieving truthful communication even in the absence of such reputation effects.

[4] Given the assumed timing of events, it is immaterial whether consumers receive signals. For simplicity, we will assume that they do not.

and precisions $\alpha = 1/\sigma_\eta^2$ and $\delta = 1/\sigma_\varepsilon^2$, respectively. Moreover, $E(\varepsilon_i \varepsilon_j) = 0$ holds for $i \neq j$. We use $\mathbf{x}$ to denote the profile of private signals across all investors. The true state, θ, becomes publicly known at the beginning of stage 1.

In stage 1, investors can produce the stage-1-good, q, at a cost $c(q) = -q$. The remaining agents can consume q, deriving utility $u(q)$, where u is increasing, concave, and satisfies the usual Inada conditions. In particular, there is a unique q^* such that $u'(q^*) = 1$. Good q is sold in a competitive market. Traders are anonymous in this market, so money is used in the exchange of q.

In stage 1 of each period, consumers have access to a lending facility operated by the CB, where they can borrow money at an interest rate $r \geq 0$. The CB keeps track of all such borrowing and loans are settled in stage 2.[5]

Finally, in stage 2, stage-0-investment implies a return of $\theta^2 k_i$ units of the non-storable stage-2-good, z. This good is traded in a competitive market in stage 2. The utility (disutility) of consumption (production) from z is linear and is denoted by $(-)z$. Those consumers that borrowed money in order to consume in stage 1 will have to produce in order to pay off these loans. The investors, who also produce in stage 1, will end up consuming in stage 2.[6]

3 The Full-Information Economy

Throughout the paper we will concentrate on the following question. What is the best way for the CB to communicate its information to the private investors? As we

[5] For simplicity, we assume that the CB makes a lump-sum transfer, τ, in the settlement market to redistribute any profits made by its lending facility.

[6] Given the linearity assumption, agents will exit stage 2 with equal money holdings. This dramatically improves tractability as it effectively simplifies our setup to an infinitely repeated version of a three-stage model.

shall see, a simple announcement will not be enough, as there are cases where the CB would prefer to make a false announcement. However, credible information revelation is possible through monetary policy. Before we study these issues, we introduce the full-information economy as a benchmark.

3.1 Efficient Allocations

Aggregate period-t welfare, $\mathcal{W}$, given by

$$\begin{aligned} \mathcal{W}(k_i, \theta) &= \frac{1}{2}\left[u(q) - q + \int_0^1 \theta^2 k_i di - \int_0^1 c(k_i)\, di\right] \\ &= \frac{1}{2}\left[u(q) - q + \theta^2 K - \int_0^1 c(k_i)\, di\right], \end{aligned} \tag{1}$$

where $K = \int_0^1 k_i di$ is the aggregate production of the investment good in stage 0. It is straightforward to verify that the efficient allocation in any given period is given by $(k_i, q) = (K, q^*)$, such that $c'(K) = \theta^2$.[7] As an example, consider the case of a quadratic cost function. We can then write

$$\begin{aligned} \mathcal{W}(k_i, K, \theta) &= \frac{1}{2}\left[u(q) - q + \int_0^1 \theta^2 k_i di - \int_0^1 \frac{k_i^2}{2} di\right] \\ &= \frac{1}{2}\left[u(q) - q + \theta^2 K - \frac{K^2}{2} - \int_0^1 \frac{(k_i - K)^2}{2} di\right]. \end{aligned} \tag{2}$$

Next, we discuss how efficient allocations can be decentralized.

3.2 Monetary Equilibrium

As private agents are not readily identifiable during stage 1, some type of record-keeping is needed for transactions to take place. In this section we discuss how full-information efficient allocations can be decentralized through monetary trade.

[7] The amount of good z produced in stage 2 is indeterminate.

Throughout we assume that money is provided exclusively by the CB. We let M denote the per capita supply of money. The growth rate of money is fixed by the CB to γ. Monetary injections are implemented through a transfer, T, in the settlement market. The CB announces its lending rate r at the end of the settlement stage. Since the transfer τ equals the CB profits from its lending facility, the net stock of money grows by $M_{+1} = M + T$, where T is such that $M_{+1} = \gamma M$. We consider a stationary equilibrium where $\phi M = \phi_+ M_+$, so that $\gamma = \phi/\phi_+$.

We use $W(k, m, l; \theta)$ to denote the discounted lifetime utility of an agent when he enters stage 2 holding k units of the investment good, m units of money, and l units of loans from the CB, given that the realized productivity shock is θ. The function $V(m)$ denotes the expected discounted lifetime utility from entering stage 0 with money holdings m. Then, $W(k, m, l; \theta)$ is defined by

$$\begin{aligned} W(k, m, l; \theta) &= \max_{z, m_{+1}} \{-z + \beta E V(m_{+1}; \theta', r_+)\} \\ & \quad s.t. \quad \phi m_{+1} = z + \theta^2 k + \phi m - \phi (1 + r) l + \phi \tau + \phi T, \end{aligned} \tag{3}$$

where z denotes the net production of the general good.[8] The first order and envelope conditions give

$$\beta E V_m = \phi, \; W_k = \theta^2, \; W_m = \phi, \; W_l = -\phi (1 + r). \tag{4}$$

The discounted lifetime utility of agents entering stage 1 with m units of money is[9]

$$\begin{aligned} V(m) &= \frac{1}{2} \max_{k_i} \left\{ -c(k_i) + E \left[\max_{q,d} -q + W(k_i, m + pq, 0, \theta) \right] \right\} \\ & \quad + \frac{1}{2} E \left[\max_{q,l, \text{ s.t. } pq \le m + l} u(q) + W(0, m - pq + l, l, \theta) \right]. \end{aligned} \tag{5}$$

[8] Notice that W does not explicitly depend on r_+ since the CB announces it once all other decisions have been taken.

[9] The following expression uses the fact that producers never borrow from the CB.

Using (4), the first order conditions for producers are

$$p\phi = 1, \text{ and} \tag{6}$$

$$c'(k_i) = \theta^2. \tag{7}$$

The expression in (6) is independent of x_i as θ is publicly known at that stage. The first order conditions for consumers are

$$u'(q) = \phi p (1+\lambda), \text{ and} \tag{8}$$

$$W_m + W_l + \lambda = 0, \tag{9}$$

where $\phi\lambda$ is the Lagrange multiplier on their budget constraint. Using (4), (6) in (9) we obtain $\lambda = r$. Then, (8) becomes

$$u'(q) = 1 + r. \tag{10}$$

Given an interest rate, r, a stationary equilibrium outcome is described by a pair (q, k_i) such that (7) and (10) hold for each i.

Using (7) and (10) it is straightforward to demonstrate that, when the true value of θ is publicly observable, the Friedman rule decentralizes the efficient allocation.

4 The Private Information Economy

We now turn to the case where the aggregate state of the economy, θ, is unknown. Both the private sector and the CB receive informative signals regarding the true value of θ. The CB maximizes expected period-t welfare given its signal, y, and precision, α. This is given by

$$\begin{aligned} \mathcal{W}(k_i, K, \theta) &= \frac{1}{2} E\left[u(q) - q + \int_0^1 \theta^2 k_i di - \int_0^1 c(k_i)\, di \mid \alpha, y\right] \\ &= \frac{1}{2} E\left[u(q) - q + \theta^2 K - \int_0^1 c(k_i)\, di \mid \alpha, y\right], \end{aligned} \tag{11}$$

where $K = \int_0^1 k_i di$ is the aggregate production of the investment good. Since c is convex, Jensen's inequality implies

$$\mathcal{W}(k_i, K, \theta) < \frac{1}{2} E\left[u(q) - q + \theta^2 K - c(K) \mid \alpha, y\right]. \tag{12}$$

Hence, the convexity of the private investors' cost functions implies that the benevolent CB would prefer to minimize the dispersion in the production of the investment good. Importantly, individual investor optimization does not take into account this effect. Analogously to the full information case, the efficient allocation is given by $(k_i, q) = (K, q^*)$, such that $c'(K) = E\left(\theta^2 | y, \alpha\right) = y^2 + \frac{1}{\alpha}$.

Since the CB's signal is informative, it is beneficial if this information reaches the private sector. How should the CB transmit its information to the private investors? We will consider two possibilities. First, the CB could make a direct "announcement," informing the private investors of both its private signal, y, and its confidence in its signal, α. Alternative, the CB could indirectly announce these values through monetary policy; i.e., through adapting the interest rate, r.[10] A main finding of our paper is that these two ways of transmitting information can have very different consequences. To see this, recall that the objective of the benevolent CB and that of an individual investor in the economy are not directly aligned. Hence, the CB might have an incentive to lie if not announcing the true value of its signal (or its precision) will lead to private investment decisions that improve ex ante social welfare. Being rational, the private investors might in turn choose to ignore such announcements. In contrast, transmission through changes in the interest rate is not "cheap talk," as it reduces welfare through creating a distortion associated with a violation of the Friedman rule. We will demonstrate that this adds a sufficient amount of "credibility"

[10] It might seem surprising that the CB can fully inform the public of a two-dimensional variable (y, α), through the use of an one-dimensional variable, r. In Section 5 we demonstrate that this is indeed possible.

that will, in turn, induce the private investors to rationally take into account the information provided by the CB.

4.1 Information Transmission Through an Announcement

Here we assume that the precision of the CB's signal, α, can take one of the two values: $\{\alpha_L, \alpha_H\}$, where $\alpha_L < \alpha_H$.[11] The probability that $\alpha = \alpha_L$ is denoted by π, and the probability that $\alpha = \alpha_H$ is $1-\pi$. The realization of the CB's signal precision is only observed by the CB. For simplicity, here we assume that the CB announces its signal, truthfully ($y_a = y$), or, equivalently, that y is publicly observable. In this case, the public (possibly untruthful) announcement concerns only the value of CB's signal's precision, α_a.

Analogously to the full information case, the discounted lifetime utility of agents when they enter stage 1 with m units of money is

$$\begin{aligned} V(m|y_a) &= \frac{1}{2}\max_{k_i}\left\{-c(k_i) + E\left[\max_{q,d} -q + W\left(k_i, m+pq, 0, \theta\right)|x_i, y_a, \alpha_a\right]\right\} \\ &+ \frac{1}{2}E\left[\max_{q,l,\text{ s.t. } pq\leq m+l} u\left(q\right) + W\left(0, m-pq+l, l, \theta\right)|y_a, \alpha_a\right]. \end{aligned} \tag{13}$$

Likewise, the first order conditions for investors are

$$\begin{aligned} p\phi &= 1, \text{ and} \\ c'\left(k_i\right) &= E\left(\theta^2|x_i, y_\alpha, \alpha_a\right). \end{aligned} \tag{14}$$

[11] All the arguments in this paper generalize to the case of an arbitrary finite number of precision values.

The expected aggregate welfare is given by (see Appendix for derivation):

$$\begin{aligned} 2\mathcal{W}(k_i, K|\alpha_a, \alpha, y) &= u(q_a) - q_a + \frac{y^2 + \frac{1}{\alpha}}{(\alpha_a+\delta)^2}\left(\alpha_a^2 y^2 + \alpha_a + 2\delta\right) - \frac{1}{2}\frac{(\alpha_a+2\delta)^2}{(\alpha_a+\delta)^4} \\ &\quad -\frac{1}{2}\alpha_a^2 y^2 \frac{y^2\left(\alpha_a^2 + 4\alpha_a\delta + 6\delta^2\right) + 6\delta^2\frac{1}{\alpha}}{(\alpha_a+\delta)^4} - \frac{(\alpha_a+2\delta)\left(y^2(\alpha_a+\delta)^2 + \delta^2\frac{1}{\alpha}\right)}{(\alpha_a+\delta)^4} \\ &\quad -\frac{2\delta y^2}{(\alpha_a+\delta)^2} - \frac{1}{2}\frac{\delta^2(4\delta-\alpha)}{\alpha(\alpha_a+\delta)^4}. \end{aligned} \tag{15}$$

The sign of the derivative of the welfare function with respect to α_a is ambiguous (depending on parameters, the welfare function can be concave or convex). However, as the following result asserts, there are cases where, if the private investors believe the CB's announcement, ex ante social welfare is increased if the CB misrepresents its confidence in its signal.

Consider the case when y is known but precision α is not. The expected welfare is given by

$$\begin{aligned} 2\mathcal{W}(k_i, K|\alpha_a, \alpha, y) &= u(q_a) - q_a + \frac{y^2 + \frac{1}{\alpha}}{(\alpha_a+\delta)^2}\left(\alpha_a^2 y^2 + \alpha_a + 2\delta\right) - \frac{1}{2}\frac{(\alpha_a+2\delta)^2}{(\alpha_a+\delta)^4} \\ &\quad -\frac{1}{2}\alpha_a^2 y^2 \frac{y^2\left(\alpha_a^2 + 4\alpha_a\delta + 6\delta^2\right) + 6\delta^2\frac{1}{\alpha}}{(\alpha_a+\delta)^4} - \frac{(\alpha_a+2\delta)\left(y^2(\alpha_a+\delta)^2 + \delta^2\frac{1}{\alpha}\right)}{(\alpha_a+\delta)^4} \\ &\quad -\frac{2\delta y^2}{(\alpha_a+\delta)^2} - \frac{1}{2}\frac{\delta^2(4\delta-\alpha)}{\alpha(\alpha_a+\delta)^4}. \end{aligned} \tag{16}$$

The first-order necessary condition is given by:

$$\begin{aligned} &(\delta+\alpha_a)^{-5}\alpha^{-1}\left(5\alpha\delta\alpha_a + 12\delta^3 + 4\alpha\delta^2 - \alpha_a^3 + \alpha\alpha_a^2 - 5\delta\alpha_a^2 - 4\delta^2\alpha_a + 4y^2\alpha\delta^3 + 2y^2\delta\alpha_a^3 - 4y^2\delta^3\alpha_a\right. \\ &\qquad \left. +4y^2\alpha\delta\alpha_a^2 + 8y^2\alpha\delta^2\alpha_a + 2y^4\alpha\delta\alpha_a^3 - 4y^4\alpha\delta^3\alpha_a + 10y^2\delta^2\alpha_a^2 + 4y^4\alpha\delta^2\alpha_a^2\right) = 0. \end{aligned}$$

Thus,

$$\begin{aligned} &5\alpha\delta\alpha_a + 12\delta^3 + 4\alpha\delta^2 - \alpha_a^3 + \alpha\alpha_a^2 - 5\delta\alpha_a^2 - 4\delta^2\alpha_a + 4y^2\alpha\delta^3 + 2y^2\delta\alpha_a^3 - 4y^2\delta^3\alpha_a \\ &\qquad + 4y^2\alpha\delta\alpha_a^2 + 8y^2\alpha\delta^2\alpha_a + 2y^4\alpha\delta\alpha_a^3 - 4y^4\alpha\delta^3\alpha_a + 10y^2\delta^2\alpha_a^2 + 4y^4\alpha\delta^2\alpha_a^2 = 0 \end{aligned}$$

must hold in the interior optimum. This leads to the following Proposition.

Proposition 1 *Suppose* $\alpha \geq \delta > 0$*. There exists an* $\varepsilon > 0$ *such that any* $\alpha_a \in (\alpha, \alpha + \varepsilon)$ *is preferred by the CB to announcing the truth.*

Proof. Since $\alpha \geq \delta > 0$, the first order condition becomes

$$\begin{aligned} \frac{\partial}{\partial \alpha_a} \mathcal{W}(k_i, K|\alpha_a, \alpha, y) \quad &| \quad_{\alpha=\alpha_a} = 12\delta^3 + 2y^2\delta\alpha^3 + 4y^2\alpha^3\delta + 8y^2\alpha^2\delta^2 \\ +2y^4\delta\alpha^4 + 10y^2\delta^2\alpha^2 + 4y^4\alpha^2\delta^2 (\alpha - \delta) \quad &> \quad 0. \end{aligned} \tag{17}$$

Thus, announcing $\alpha = \alpha_a$ is not optimal. Moreover, since $\frac{\partial}{\partial \alpha_a} \mathcal{W}(k_i, K|\alpha_a, \alpha, y) \mid_{\alpha=\alpha_a} > 0$, the CB prefers to announce an α_a which is higher than the true α. ■

Example 1 *Let* $\delta = 70$*,* $y = 0.05$*,* $\alpha_i = 60$*, and* $\alpha_j = 150$*. Then, welfare under truth-telling for the CB with* $\alpha = 60$ *is* $\mathcal{W}(k_i, K|\alpha = 60, \alpha_a = 60, y) = 0.000018$*. On the other hand, if the CB reports* $\alpha_a = 150$*, its welfare increases to* $\mathcal{W}(k_i, K|\alpha = 60, \alpha_a = 150, y) = 0.000074$*. Thus, the CB with* $\alpha = 60$ *prefers to announce* $\alpha_a = 150$ *and set* $r(\alpha_a) = 0$*.*

In other words, depending on the parameter values, the benevolent CB has an incentive to misrepresent its confidence in its information. A CB with α_H might prefer to announce α_L, where $\alpha_L < \alpha_H$, or vice versa.[12] This raises the question of whether it is possible for the CB to use a *costly* message in order to communicate its confidence, α, credibly. We investigate this next.

4.2 Transmission Through Monetary Policy

Here we still consider the case where the precision of the CB's signal, α, can take one of the two values: $\{\alpha_L, \alpha_H\}$, with respective probabilities π and $1 - \pi$. We again

[12] It is easy to show that the CB will never misrepresent its confidence when $\delta = 0$.

assume that the CB announces y_a truthfully. Thus, information transmission consists of an announcement, α_a, about the value of CB's signal's precision, α.

Here we assume that, in order to transmit its information, the CB chooses a monetary policy rule $r(\alpha)$. The equilibrium conditions are derived as before, but adapted to include this information. They are given by

$$\begin{aligned} c'(k_i(x_i, y_a, \alpha_a)) &= E\left(\theta^2 | x_i, y_a, \alpha_a\right), \text{and} \\ u'(q(x_i, y_a, \alpha_a)) &= 1 + r(\alpha_a). \end{aligned} \tag{18}$$

As before, the problem of the CB is to maximize

$$\mathcal{W}(K, \theta) = \frac{1}{2} E\left[u(q) - q + \theta^2 K - \frac{K^2}{2} - \int \frac{(k_i - K)^2}{2} \mid \alpha, y\right], \tag{19}$$

subject to the above equilibrium equations for individual investors. In addition, in order for monetary policy to *credibly* transmit information about the CB's confidence, a set of incentive compatibility constraints must hold. More precisely, we require

$$\mathcal{W}(K | r(\alpha), \alpha) \geq \mathcal{W}(K | r(\alpha_a), \alpha) \text{ for } \alpha_a \neq \alpha. \tag{20}$$

To ease exposition, consider the case when the CB with $\alpha = \alpha_L$ prefers to mimic the CB with $\alpha = \alpha_H$, $\alpha_L < \alpha_H$. (The other case is isomorphic). Clearly, the incentive compatibility constraint in this case does not bind for the state where $\alpha = \alpha_H$. However, when $\alpha = \alpha_L$, the CB would have an incentive to misrepresent its confidence in its information. Thus, the incentive compatibility constraint must bind; i.e.,

$$\mathcal{W}(K | r(\alpha_H), \alpha_L) = \mathcal{W}(K | r(\alpha_L), \alpha_L). \tag{21}$$

Setting $r(\alpha_L) = 0$, we can obtain the corresponding interest rate, $r(\alpha_H)$, implicitly as the solution to

$$\mathcal{W}(K | r(\alpha_H), \alpha_L) = \mathcal{W}(K | 0, \alpha_L). \tag{22}$$

We must also verify that

$$\mathcal{W}(K|r(\alpha_H), \alpha_H) \geq \mathcal{W}(K|0, \alpha_H). \tag{23}$$

In other words, inflating in order to reveal that $\alpha = \alpha_H$ is preferred by a CB with $\alpha = \alpha_H$ to the alternative of not inflating and announcing $\alpha = \alpha_L$. In addition, the CB with $\alpha = \alpha_H$ must prefer the corresponding outcome to the case where there is no inflationary distortion, but private agents take

$$\overline{\alpha} = \pi\alpha_L + (1-\pi)\alpha_H.$$

In other words, we need to ensure that

$$\mathcal{W}(K|r(\alpha_H), \alpha_H) \geq \mathcal{W}(K|r(\overline{\alpha}), \alpha_H), \tag{24}$$

where $r(\overline{\alpha}) = 0$. We have the following.

Proposition 2 *Suppose that the precision of the public signal, α, can take one of the two values, α_i or α_j, $\alpha_i \neq \alpha_j$. Suppose that $\frac{1}{2} > y^2\delta$ and $\alpha_H > 3\delta$, and let $\pi_H \to 0$. Then, to credibly communicate α, the CB with $\alpha = \alpha_i$ prefers to set $r(\alpha_i) > 0$, while the CB with $\alpha = \alpha_j$ sets $r(\alpha_j) = 0$. The interest rate, $\widehat{r}(\alpha_i)$, is implicitly given by $\mathcal{W}(K|r(\alpha_i), \alpha_j) = \mathcal{W}(K|0, \alpha_j)$.*

Proof. When y is known but α is not, expected welfare is given by

$$\begin{aligned}
2\mathcal{W}(k_i, K|\alpha_a, \alpha, y) &= u(q_a) - q_a + \frac{y^2 + \frac{1}{\alpha}}{(\alpha_a+\delta)^2}\left(\alpha_a^2 y^2 + \alpha_a + 2\delta\right) - \frac{1}{2}\frac{(\alpha_a+2\delta)^2}{(\alpha_a+\delta)^4} \\
&\quad -\frac{1}{2}\alpha_a^2 y^2 \frac{y^2\left(\alpha_a^2 + 4\alpha_a\delta + 6\delta^2\right) + 6\delta^2\frac{1}{\alpha}}{(\alpha_a+\delta)^4} - \frac{(\alpha_a+2\delta)\left(y^2(\alpha_a+\delta)^2 + \delta^2\frac{1}{\alpha}\right)}{(\alpha_a+\delta)^4} \\
&\quad -\frac{2\delta y^2}{(\alpha_a+\delta)^2} - \frac{1}{2}\frac{\delta^2(4\delta-\alpha)}{\alpha(\alpha_a+\delta)^4}.
\end{aligned} \tag{25}$$

Then

$$\frac{\partial \mathcal{W}(\alpha_a, \alpha, y)}{\partial \alpha_a} = (\delta + \alpha_a)^{-5} \alpha^{-1}$$
$$\big(5\alpha\delta\alpha_a + 12\delta^3 + 4\alpha\delta^2 - \alpha_a^3 + \alpha\alpha_a^2 - 5\delta\alpha_a^2 - 4\delta^2\alpha_a + 4y^2\alpha\delta^3 + 2y^2\delta\alpha_a^3 - 4y^2\delta^3\alpha_a$$
$$+4y^2\alpha\delta\alpha_a^2 + 8y^2\alpha\delta^2\alpha_a + 2y^4\alpha\delta\alpha_a^3 - 4y^4\alpha\delta^3\alpha_a + 10y^2\delta^2\alpha_a^2 + 4y^4\alpha\delta^2\alpha_a^2\big).$$

It is sufficient to show that $\frac{\partial \mathcal{W}(\alpha_a,\alpha_H,y)}{\partial \alpha_a} |_{\alpha_a=\alpha_H} > \frac{\partial \mathcal{W}(\alpha_a,\alpha_L,y)}{\partial \alpha_a} |_{\alpha_a=\alpha_H}$, where $\alpha_H = \alpha_L + \varepsilon$. We have that $\frac{\partial \mathcal{W}(\alpha_a,\alpha_H,y)}{\partial \alpha_a} |_{\alpha_a=\alpha_H}$ is given by

$$(\delta + \alpha_H)^{-5} \alpha_H^{-1} \big(12\delta^3 + 2y^2\delta\alpha_H^3 + 4y^2\delta\alpha_H^3 + 8y^2\alpha_H^2\delta^2 + 2y^4\delta\alpha_H^4 + 10y^2\delta^2\alpha_H^2 + 4y^4\delta^2\alpha_H^3 - 4y^4\alpha_H^2\delta^3\big), \tag{26}$$

while $\frac{\partial \mathcal{W}(\alpha_a,\alpha_L,y)}{\partial \alpha_a} |_{\alpha_a=\alpha_H}$ is given by

$$(\delta + \alpha_H)^{-5} (\alpha_H - \varepsilon)^{-1}$$
$$\left(\begin{array}{c} 5(\alpha_H - \varepsilon)\delta\alpha_H + 12\delta^3 + 4(\alpha_H - \varepsilon)\delta^2 - \alpha_H^3 \\ +(\alpha_H - \varepsilon)\alpha_H^2 - 5\delta\alpha_H^2 - 4\delta^2\alpha_H + 4y^2(\alpha_H - \varepsilon)\delta^3 + 2y^2\delta\alpha_H^3 \end{array} \right. \tag{27}$$
$$\left. \begin{array}{c} -4y^2\delta^3\alpha_H + 4y^2(\alpha_H - \varepsilon)\delta\alpha_H^2 + 8y^2(\alpha_H - \varepsilon)\delta^2\alpha_H + 2y^4 \\ (\alpha_H - \varepsilon)\delta\alpha_H^3 - 4y^4(\alpha_H - \varepsilon)\delta^3\alpha_H + 10y^2\delta^2\alpha_H^2 + 4y^4(\alpha_H - \varepsilon)\delta^2\alpha_H^2 \end{array} \right), \tag{28}$$

where we write α_L as $\alpha_H - \varepsilon$. Then, $\frac{\partial \mathcal{W}(\alpha_a,\alpha_H,y)}{\partial \alpha_a} |_{\alpha_a=\alpha_H} - \frac{\partial \mathcal{W}(\alpha_a,\alpha_L,y)}{\partial \alpha_a} |_{\alpha_a=\alpha_H}$ is given by

$$(\varepsilon - \alpha_H)^{-1} (\delta + \alpha_H)^{-5} \alpha_H^{-1} \big(12\delta^3 - \alpha_H^3 - 5\delta\alpha_H^2 - 4\delta^2\alpha_H + 2y^2\delta\alpha_H^3 - 4y^2\delta^3\alpha_H + 10y^2\delta^2\alpha_H^2\big)\varepsilon. \tag{29}$$

Since $\varepsilon > 0$ is "small", we have that $\varepsilon - \alpha_H < 0$. Thus, we must show that

$$12\delta^3 - \alpha_H^3 - 5\delta\alpha_H^2 - 4\delta^2\alpha_H + 2y^2\delta\alpha_H^3 - 4y^2\delta^3\alpha_H + 10y^2\delta^2\alpha_H^2 < 0, \tag{30}$$

or, equivalently,

$$\alpha_H^3\big(1 - 2y^2\delta\big) + \alpha_H^2\delta\big(5 - 10y^2\delta\big) + 4\delta^2\big(\alpha_H\big(1 + y^2\delta\big) - 3\delta\big) > 0. \tag{31}$$

Sufficient conditions for the above inequality are

$$\begin{aligned} 1 - 2y^2\delta &> 0 \\ 5 - 10y^2\delta &> 0 \\ \alpha_H\left(1 + y^2\delta\right) - 3\delta &> 0 \end{aligned} \tag{32}$$

or, simplifying,

$$\begin{aligned} \frac{1}{2} &> y^2\delta \\ \alpha_H &> 3\delta. \end{aligned} \tag{33}$$

■

Example 2 *Parameter values are as in the Example 1 above. Moreover, let* $\pi(\alpha = 60) = 0.95$*. We know that the CB with* $\alpha = 60$ *has an incentive to report* $\alpha_a = 150$*. The question is whether the CB with* $\alpha = 150$ *prefers to costly but credibly communicate the true value of its precision.*[13] *To prevent the CB with* $\alpha = 60$ *from communicating the value* $\alpha = 150$*, the CB must invoke the cost* r*, where* r *solves*

$$u(q(r)) - q(r) - u(q^*) + q^* = -0.000056. \tag{34}$$

Since the above difference is negative, such an r *exists. To see whether the CB with* $\alpha = 150$ *chooses credible but costly communication we first check that welfare under costly communication is higher than welfare under not inflating and being taken for an* $\alpha = 60$ *type:*

$$\mathcal{W}(k_i, K|r(\alpha), \alpha = \alpha_a = 150) - \mathcal{W}(k_i, K|0, \alpha = 150, \alpha_a = 60) = 0.169013\ 10^{-5}. \tag{35}$$

[13] Note that the welfare of the CB with $\alpha = 150$ when it is taken to be the CB with $\alpha = 60$ is given by $\mathcal{W}(k_i, K|\alpha = 150, \alpha_a = 60, y) = -\ 0.00004231761264$.

Second, we check that welfare under costly communication exceeds welfare under "pooling," whereby private investors use their priors on α and the CB sets $r = 0$:

$$\mathcal{W}(k_i, K|r(\alpha), \alpha = \alpha_a = 150) - \mathcal{W}(k_i, K|0, \alpha = 150, \alpha_a = \overline{\alpha}) = 0.674682\ 10^{-7}. \quad (36)$$

The above Propositions taken together imply that there exist parameter values under which in order to transmit information to investors, the CB must create some inflation. In fact, incentive constraints put a lower bound on the inflation level that must be tolerated in order for the information transmission to be credible. Otherwise, like a simple announcement, investors might rationally ignore the supplied information, treating it as "cheap talk."

Thus, at least in some cases, monetary policy achieves a credible transmission of information by the CB. A natural question is then whether the CB can always communicate its signal, as well as its confidence in this information, through monetary policy. in other words, the question is whether the one-dimensional set of feasible monetary policies is "rich" enough to communicate all possible values of the two-dimensional set consisting of the CB's signal and precision. In the next section we demonstrate that the answer to this question is affirmative. In other words, in a well defined mathematical sense, monetary policy is nothing but a "translation" of a message revealing the CB's information and confidence. As we argued here, this message is credible since it involves a real cost. Thus, unlike a simple announcement, it is not rationally treated as "cheap talk."

5 An Equivalence Result

Here we demonstrate how the CB can in principle use monetary policy in order to convey *both* the realized value of its signal and its confidence in the signal through manipulating the interest rate.

To study this issue, consider again the case where the precision of the public signal, α, can take one of the two values: $\{\alpha_L, \alpha_H\}$, where $\alpha_L < \alpha_H$. The probability that $\alpha = \alpha_L$ is denoted by π. As before, the CB receives a signal y about the value of θ. In addition, we assume that the CB knows the realization of α. The CB can choose to convey the value of y and α to the public through monetary policy. This can be accomplished via a rule that takes the following form:

$$r = h(y, \alpha), \tag{37}$$

with $h(y, \alpha) = 0$, if the CB does not reveal its information in state (y, α). Can the CB use monetary policy in order to reveal its information about *both* its signal and that signal's precision? Put differently, is the (expanded) information set of the CB equivalent to the set of monetary policies in can apply? One contribution of our paper is to formulate the above as a mathematical question. To give an affirmative answer to this question, we need to demonstrate that there exists a homeomorphism between these two sets.

Since the CB needs to signal both y and α to the public, it must be the case that $h(y, \alpha_L) \neq h(y', \alpha_H)$ for any y, y'. One candidate policy for $i \neq j \in \{H, L\}$ is defined as follows[14]

$$h(y, \alpha) = \begin{cases} h(y) \text{ if } \alpha = \alpha_i \\ h(y) + \widehat{r} \text{ if } \alpha = \alpha_j \end{cases}, \tag{38}$$

Indeed, for this policy, we have the following for any $\Delta > \widehat{r} > 0$.

Proposition 3 *The function $h(y, \alpha)$ defined above, $h(y, \alpha) : \mathbb{R} \times \{\alpha_L, \alpha_H\} \rightarrow (0, \Delta) \cup (\widehat{r}, \widehat{r} + \Delta)$ is a homeomorphism.*

[14] Recall that $\widehat{r}$ stands for the minimum level of the interest rate that makes the corresponding information revelation credible.

The proof, which appears in the Appendix, readily generalizes to a countable set of possible precision values. The above Proposition demonstrate the (topological) equivalence between the set describing the information potentially held by the CB and the set of feasible and credible monetary policies at the CB's disposal. This implies that, at the cost of imposing a distortion associated with some inflation, the CB can use monetary policy in order to reveal both its signal about fundamentals and its confidence in that signal to the private sector.

6 Discussion

We studied a model where uncertainty can lead to a discrepancy between investors' beliefs and true fundamentals. A main finding of our paper identifies monetary policy as essentially the only tool at a CB's disposal that can lead to credible information transmission. Since the objective of the benevolent CB and that of an individual investor in our economy are not directly aligned, the CB might have an incentive to misrepresent its information in order to improve ex ante social welfare. Being rational, the private investors will ignore such announcements. In contrast, information transmission through changes in the interest rate is not "cheap talk," as it reduces welfare through creating a distortion associated with a violation of the Friedman rule. We showed that this adds a sufficient amount of "credibility," thus, inducing private investors to rationally take into account the information provided by the CB. Finally, we formalized the notion that monetary policy is equivalent to (a translation of) information revelation by the CB.

7 Appendix

7.1 Derivation of welfare

We derive expected welfare assuming that the CB announces its signal y truthfully. We ask whether it may have an incentive to announce $\alpha_a \neq \alpha$. To compute $\mathcal{W}(k_i, K|\alpha_a, \alpha, y)$, we decompose and calculate each terms separately before adding them up. When the CB announces (α_a, y), investor i invests $k_{i,a} = k(\alpha_a, y, x_i)$. The expected aggregate welfare is given by:

$$\mathcal{W}(k_i, K|\alpha_a, \alpha, y) = \frac{1}{2}E\left[u(q_a) - q_a + \theta^2 K - \frac{K^2}{2} - \int \frac{(k_i - K)^2}{2} \mid \alpha, y\right]. \quad (39)$$

The first-order condition for investors gives

$$k_i = E\left[\theta^2 \mid \alpha_a, y, x_i\right] = \left(\frac{\alpha_a y + \delta x_i}{\alpha_a + \delta}\right)^2 + \frac{1}{\alpha_a + \delta}. \quad (40)$$

Note that $\frac{\partial k_i}{\partial \alpha_a} - (\delta + \alpha_a)^{-3}(\delta + \alpha_a + 2\delta[x_i - y][y\alpha_a + \delta x_i])$ and so for $x_i > y$, $\frac{\partial k_i}{\partial \alpha_a} < 0$. We then have

$$\begin{aligned}
K &= \int k_i di = \int \left(\frac{\alpha_a y + \delta x_i}{\alpha_a + \delta}\right)^2 di + \frac{1}{\alpha_a + \delta} \\
&= \int \frac{\alpha_a^2 y^2 + 2\alpha_a \delta y(\theta + \varepsilon_i) + \delta^2\left(\theta^2 + 2\theta\varepsilon_i + \varepsilon_i^2\right)}{(\alpha_a + \delta)^2} di + \frac{1}{\alpha_a + \delta} \\
&= \frac{\alpha_a^2 y^2 + 2\alpha_a \delta y\theta + \delta^2\theta^2 + \delta}{(\alpha_a + \delta)^2} + \frac{1}{\alpha_a + \delta} \\
&= \frac{(\alpha_a y + \delta\theta)^2}{(\alpha_a + \delta)^2} + \frac{\alpha_a + 2\delta}{(\alpha_a + \delta)^2}. \quad (41)
\end{aligned}$$

Thus,

$$
\begin{aligned}
E\left[f\left(K\right)\mid\alpha,y\right] &= E\left[\theta^{2}K\mid\alpha,y\right]=E\left[\theta^{2}\frac{\left(\alpha_{a}y+\delta\theta\right)^{2}}{\left(\alpha_{a}+\delta\right)^{2}}+\theta^{2}\frac{\alpha_{a}+2\delta}{\left(\alpha_{a}+\delta\right)^{2}}\mid\alpha,y\right] \\
&= E\left[\frac{\alpha_{a}^{2}y^{2}\theta^{2}+2\alpha_{a}\delta y\theta^{3}+\delta^{2}\theta^{4}}{\left(\alpha_{a}+\delta\right)^{2}}+\theta^{2}\frac{\alpha_{a}+2\delta}{\left(\alpha_{a}+\delta\right)^{2}}\mid\alpha,y\right] \\
&= \frac{\alpha_{a}^{2}y^{2}}{\left(\alpha_{a}+\delta\right)^{2}}\left(y^{2}+\frac{1}{\alpha}\right)+\frac{\alpha_{a}+2\delta}{\left(\alpha_{a}+\delta\right)^{2}}\left(y^{2}+\frac{1}{\alpha}\right) \\
&= \frac{y^{2}+\frac{1}{\alpha}}{\left(\alpha_{a}+\delta\right)^{2}}\left(\alpha_{a}^{2}y^{2}+\alpha_{a}+2\delta\right). \qquad (42)
\end{aligned}
$$

Now,

$$
\begin{aligned}
E\left[K^{2}\mid\alpha,y\right] &= E\left[\left(\frac{\left(\alpha_{a}y+\delta\theta\right)^{2}}{\left(\alpha_{a}+\delta\right)^{2}}+\frac{\alpha_{a}+2\delta}{\left(\alpha_{a}+\delta\right)^{2}}\right)^{2}\mid\alpha,y\right] \\
&= E\left[\frac{\left(\alpha_{a}y+\delta\theta\right)^{4}}{\left(\alpha_{a}+\delta\right)^{4}}+2\frac{\left(\alpha_{a}y+\delta\theta\right)^{2}\left(\alpha_{a}+2\delta\right)}{\left(\alpha_{a}+\delta\right)^{4}}+\frac{\left(\alpha_{a}+2\delta\right)^{2}}{\left(\alpha_{a}+\delta\right)^{4}}\mid\alpha,y\right] \\
&= E\left[\frac{\alpha_{a}^{4}y^{4}+\delta^{4}\theta^{4}+4\alpha_{a}y\delta^{3}\theta^{3}+4\alpha_{a}^{3}y^{3}\delta\theta+6\alpha_{a}^{2}y^{2}\delta^{2}\theta^{2}}{\left(\alpha_{a}+\delta\right)^{4}}+2\frac{\left(\alpha_{a}y+\delta\theta\right)^{2}\left(\alpha_{a}+2\delta\right)}{\left(\alpha_{a}+\delta\right)^{4}}\right. \\
&\quad \left.+\frac{\left(\alpha_{a}+2\delta\right)^{2}}{\left(\alpha_{a}+\delta\right)^{4}}\mid\alpha,y\right] \\
&= \frac{\alpha_{a}^{4}y^{4}+4\alpha_{a}^{3}y^{4}\delta+6\alpha_{a}^{2}y^{2}\delta^{2}\left(y^{2}+\frac{1}{\alpha}\right)}{\left(\alpha_{a}+\delta\right)^{4}}+2\frac{\left(\alpha_{a}+2\delta\right)\left(\alpha_{a}^{2}y^{2}+2\alpha_{a}\delta y^{2}+\delta^{2}y^{2}+\delta^{2}\frac{1}{\alpha}\right)}{\left(\alpha_{a}+\delta\right)^{4}} \\
&\quad +\frac{\left(\alpha_{a}+2\delta\right)^{2}}{\left(\alpha_{a}+\delta\right)^{4}} \\
&= \alpha_{a}^{2}y^{2}\frac{\alpha_{a}^{2}y^{2}+4\alpha_{a}y^{2}\delta+6y^{2}\delta^{2}+6\delta^{2}\frac{1}{\alpha}}{\left(\alpha_{a}+\delta\right)^{4}}+2\frac{\left(\alpha_{a}+2\delta\right)\left(y^{2}\left(\alpha_{a}+\delta\right)^{2}+\delta^{2}\frac{1}{\alpha}\right)}{\left(\alpha_{a}+\delta\right)^{4}} \\
&\quad +\frac{\left(\alpha_{a}+2\delta\right)^{2}}{\left(\alpha_{a}+\delta\right)^{4}}. \qquad (43)
\end{aligned}
$$

Then,

$$E\left[f\left(K\right)d_i-\frac{K^2}{2}|\alpha,y\right] = \frac{y^2+\frac{1}{\alpha}}{\left(\alpha_a+\delta\right)^2}\left(\alpha_a^2y^2+\alpha_a+2\delta\right)-\frac{1}{2}\frac{\left(\alpha_a+2\delta\right)^2}{\left(\alpha_a+\delta\right)^4}$$
$$-\frac{1}{2}\alpha_a^2y^2\frac{y^2\left(\alpha_a^2+4\alpha_a\delta+6\delta^2\right)+6\delta^2\frac{1}{\alpha}}{\left(\alpha_a+\delta\right)^4}$$
$$-\frac{\left(\alpha_a+2\delta\right)\left(y^2\left(\alpha_a+\delta\right)^2+\delta^2\frac{1}{\alpha}\right)}{\left(\alpha_a+\delta\right)^4}. \tag{44}$$

Next, we compute the expression for $E\left[\int\left(k_i-K\right)^2di\mid\alpha,y\right]$. We have

$$E\left[\int\left(\frac{\left(\alpha_ay+\delta x_i\right)^2}{\left(\alpha_a+\delta\right)^2}+\frac{1}{\alpha_a+\delta}-\frac{\left(\alpha_ay+\delta\theta\right)^2}{\left(\alpha_a+\delta\right)^2}-\frac{\alpha_a+2\delta}{\left(\alpha_a+\delta\right)^2}\right)^2di\mid\alpha,y\right]$$
$$=E\left[\int\left(\frac{\left(\alpha_ay+\delta x_i\right)^2-\left(\alpha_ay+\delta\theta\right)^2}{\left(\alpha_a+\delta\right)^2}-\frac{\delta}{\left(\alpha_a+\delta\right)^2}\right)^2di\mid\alpha,y\right]$$
$$=E\left[\int\left(\frac{\left(\alpha_ay+\delta x_i+\alpha_ay+\delta\theta\right)\left(\alpha_ay+\delta x_i-\alpha_ay-\delta\theta\right)}{\left(\alpha_a+\delta\right)^2}-\frac{\delta}{\left(\alpha_a+\delta\right)^2}\right)^2di\mid\alpha,y\right]$$

$$= E\left[\int\left(\frac{(2\alpha_a y+\delta(x_i+\theta))\,\delta(x_i-\theta)}{(\alpha_a+\delta)^2}-\frac{\delta}{(\alpha_a+\delta)^2}\right)^2 di \mid \alpha, y\right]$$

$$= E\left[\int\left(\frac{(2\alpha_a y+\delta(\theta+\varepsilon_i+\theta))\,\delta(\theta+\varepsilon_i-\theta)}{(\alpha_a+\delta)^2}-\frac{\delta}{(\alpha_a+\delta)^2}\right)^2 di \mid \alpha, y\right]$$

$$= E\left[\int\left(\frac{2\alpha_a y\delta\varepsilon_i+\delta^2(2\theta+\varepsilon_i)\,\varepsilon_i}{(\alpha_a+\delta)^2}-\frac{\delta}{(\alpha_a+\delta)^2}\right)^2 di \mid \alpha, y\right]$$

$$= E\left[\int\left(\frac{2\alpha_a y\delta\varepsilon_i+2\delta^2\theta\varepsilon_i+\delta^2\varepsilon_i^2}{(\alpha_a+\delta)^2}-\frac{\delta}{(\alpha_a+\delta)^2}\right)^2 di \mid \alpha, y\right]$$

$$= E\left[\int\left(\frac{\left[\left(2\alpha_a y\delta+2\delta^2\theta\right)\varepsilon_i+\delta^2\varepsilon_i^2\right]^2}{(\alpha_a+\delta)^4}-2\frac{2\alpha_a y\delta^2\varepsilon_i+2\delta^3\theta\varepsilon_i+\delta^3\varepsilon_i^2}{(\alpha_a+\delta)^4}+\frac{\delta^2}{(\alpha_a+\delta)^4}\right) di \mid \alpha, y\right]$$

$$= E\left[\frac{\left(2\alpha_a y\delta+2\delta^2\theta\right)^2\frac{1}{\delta}}{(\alpha_a+\delta)^4}-2\frac{\delta^2}{(\alpha_a+\delta)^4}+\frac{\delta^2}{(\alpha_a+\delta)^4}\mid \alpha, y\right]$$

$$= E\left[\frac{4\alpha_a^2y^2\delta+8\alpha_a y\delta^2\theta+4\delta^3\theta^2}{(\alpha_a+\delta)^4}-\frac{\delta^2}{(\alpha_a+\delta)^4}\mid \alpha, y\right]$$

$$= \frac{4\alpha_a^2y^2\delta+8\alpha_a y^2\delta^2+4\delta^3y^2+4\delta^3\frac{1}{\alpha}}{(\alpha_a+\delta)^4}-\frac{\delta^2}{(\alpha_a+\delta)^4}$$

$$= 4\delta y^2\frac{\alpha_a^2+2\alpha_a\delta+\delta^2}{(\alpha_a+\delta)^4}+\frac{4\delta^3}{\alpha(\alpha_a+\delta)^4}-\frac{\delta^2}{(\alpha_a+\delta)^4}$$

$$= \frac{4\delta y^2}{(\alpha_a+\delta)^2}+\delta^2\frac{4\delta-\alpha}{\alpha(\alpha_a+\delta)^4}. \tag{45}$$

Welfare is, thus, given by

$$\begin{aligned}2\mathcal{W}(k_i,K|\alpha_a,\alpha,y) \;=\;& u(q_a)-q_a+\frac{y^2+\frac{1}{\alpha}}{(\alpha_a+\delta)^2}\left(\alpha_a^2y^2+\alpha_a+2\delta\right)-\frac{1}{2}\frac{(\alpha_a+2\delta)^2}{(\alpha_a+\delta)^4}\\ &-\frac{1}{2}\alpha_a^2y^2\frac{y^2\left(\alpha_a^2+4\alpha_a\delta+6\delta^2\right)+6\delta^2\frac{1}{\alpha}}{(\alpha_a+\delta)^4}-\frac{(\alpha_a+2\delta)\left(y^2(\alpha_a+\delta)^2+\delta^2\frac{1}{\alpha}\right)}{(\alpha_a+\delta)^4}\\ &-\frac{2\delta y^2}{(\alpha_a+\delta)^2}-\frac{1}{2}\frac{\delta^2(4\delta-\alpha)}{\alpha(\alpha_a+\delta)^4}.\end{aligned} \tag{46}$$

7.2 Derivation of welfare with both y and α unknown

To compute $\mathcal{W}(k_i, K|r(\alpha_a, y_a), \alpha, y)$, we again decompose and calculate each terms separately before adding them up. When the CB announces (α_a, y_a), investor i invests $k_{i,a} = k(\alpha_a, y_a, x_i)$. Assuming a quadratic cost function: $c(k) = k^2/2$, and a production function $f(k) = \theta^2 k_i$, welfare is given by

$$\mathcal{W}(k_i, K|\alpha, y, y_a) = \frac{1}{2}E\left[u(q_a) - q_a + \theta^2 K - \frac{K^2}{2} - \int \frac{(k_i - K)^2}{2} \mid y, \alpha\right]. \tag{47}$$

The first-order condition for producers gives

$$k_i = E\left[\theta^2 \mid x_i, y\right] = \left(\frac{\alpha_a y_a + \delta x_i}{\alpha_a + \delta}\right)^2 + \frac{1}{\alpha_a + \delta}. \tag{48}$$

Then, we have

$$K = \int k_i di = \frac{(\alpha_a y_a + \delta\theta)^2}{(\alpha_a + \delta)^2} + \frac{\alpha_a + 2\delta}{(\alpha_a + \delta)^2}, \tag{49}$$

and

$$\begin{aligned}
E[f(K) \mid y] &= E\left[\theta^2 K \mid y\right] = E\left[\theta^2 \frac{(\alpha_a y_a + \delta\theta)^2}{(\alpha_a + \delta)^2} + \theta^2 \frac{\alpha_a + 2\delta}{(\alpha_a + \delta)^2} \mid y\right] \\
&= E\left[\frac{\alpha_a^2 y_a^2 \theta^2 + 2\alpha_a \delta y_a \theta^3 + \delta^2\theta^4}{(\alpha_a + \delta)^2} + \theta^2 \frac{\alpha_a + 2\delta}{(\alpha_a + \delta)^2} \mid y\right] \\
&= \frac{\alpha_a^2 y_a^2 \left(y^2 + \frac{1}{\alpha}\right)}{(\alpha_a + \delta)^2} + \frac{\alpha_a + 2\delta}{(\alpha_a + \delta)^2}\left(y^2 + \frac{1}{\alpha}\right) \\
&= \frac{y^2 + \frac{1}{\alpha}}{(\alpha_a + \delta)^2}\left(\alpha_a^2 y_a^2 + \alpha_a + 2\delta\right) \\
&= \frac{\alpha y^2 + 1}{\alpha(\alpha_a + \delta)^2}\left(\alpha_a^2 y_a^2 + \alpha_a + 2\delta\right).
\end{aligned} \tag{50}$$

Now,

$$
\begin{aligned}
E\left[K^2 \mid y\right] &= E\left[\left(\frac{(\alpha_a y_a + \delta\theta)^2}{(\alpha_a+\delta)^2} + \frac{\alpha_a + 2\delta}{(\alpha_a+\delta)^2}\right)^2 \mid y\right] \\
&= E\left[\frac{(\alpha_a y_a + \delta\theta)^4}{(\alpha_a+\delta)^4} + 2\frac{(\alpha_a y_a + \delta\theta)^2(\alpha_a + 2\delta)}{(\alpha_a+\delta)^4} + \frac{(\alpha_a + 2\delta)^2}{(\alpha_a+\delta)^4} \mid y\right] \\
&= E\left[\frac{\theta^4\delta^4 + y_a^4\alpha_a^4 + 4y_a\theta^3\delta^3\alpha_a + 4y_a^3\theta\delta\alpha_a^3 + 6y_a^2\theta^2\delta^2\alpha_a^2}{(\alpha_a+\delta)^4} + 2\frac{(\alpha_a y_a + \delta\theta)^2(\alpha_a + 2\delta)}{(\alpha_a+\delta)^4}\right. \\
&\quad \left. + \frac{(\alpha_a + 2\delta)^2}{(\alpha_a+\delta)^4} \mid y\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{y_a^4\alpha_a^4 + 4y_a^3 y\delta\alpha_a^3 + 6y_a^2\delta^2\alpha_a^2\left(y^2 + \frac{1}{\alpha}\right)}{(\alpha_a+\delta)^4} + 2\frac{(\alpha_a + 2\delta)\left(\alpha_a^2 y_a^2 + 2\alpha_a\delta y y_a + \delta^2 y^2 + \delta^2\frac{1}{\alpha}\right)}{(\alpha_a+\delta)^4} \\
&\quad + \frac{(\alpha_a + 2\delta)^2}{(\alpha_a+\delta)^4} \\
&= y_a^2\alpha_a^2\frac{y_a^2\alpha_a^2 + 4y_a y\delta\alpha_a + 6\delta^2 y^2 + 6\delta^2\frac{1}{\alpha}}{(\alpha_a+\delta)^4} + 2\frac{(\alpha_a + 2\delta)\left((\alpha_a y_a + \delta y)^2 + \delta^2\frac{1}{\alpha}\right)}{(\alpha_a+\delta)^4} \\
&\quad + \frac{(\alpha_a + 2\delta)^2}{(\alpha_a+\delta)^4}. \qquad (51)
\end{aligned}
$$

Then,

$$
\begin{aligned}
E\left[f(K)\,d_i - \frac{K^2}{2}|\alpha, y\right] &= \frac{y^2 + \frac{1}{\alpha}}{(\alpha_a+\delta)^2}\left(\alpha_a^2 y_a^2 + \alpha_a + 2\delta\right) - \frac{1}{2}\frac{(\alpha_a + 2\delta)^2}{(\alpha_a+\delta)^4} \\
&\quad - \frac{1}{2}y_a^2\alpha_a^2\frac{y_a^2\alpha_a^2 + 4y_a y\delta\alpha_a + 6\delta^2 y^2 + 6\delta^2\frac{1}{\alpha}}{(\alpha_a+\delta)^4} \\
&\quad - \frac{(\alpha_a + 2\delta)\left((\alpha_a y_a + \delta y)^2 + \delta^2\frac{1}{\alpha}\right)}{(\alpha_a+\delta)^4}. \qquad (52)
\end{aligned}
$$

7.3 The Equivalence proof

Here we prove:

Proposition 4 *Let* $h : \mathbb{R} \rightarrow (0, \Delta)$ *be a homeomorphism where* $\Delta > 0$*. Then, the function* $H(y, \alpha) : \mathbb{R} \times \{L, H\} \rightarrow (0, \Delta) \cup (\widehat{r}, \widehat{r} + \Delta)$ *given by*

$$H(y, \alpha) = \begin{cases} h(y) & \text{if } \alpha = \alpha_L \\ h(y) + \widehat{r} & \text{if } \alpha = \alpha_H \end{cases}, \tag{53}$$

is a homeomorphism.

Proof. We impose the usual topology on $\mathbb{R}$, and the usual subspace topologies on $(0, \Delta)$ and on $(0, \Delta) \cup (\widehat{r}, \widehat{r} + \Delta)$. We impose the discrete topology on $\{\alpha_L, \alpha_H\}$. The resulting topology on $\mathbb{R} \times \{\alpha_L, \alpha_H\}$ is the product topology. We need to show that $H(y, \alpha)$ satisfies all the conditions for a homeomorphism.

Claim 1: $H(y, \alpha)$ is injective.

We need to show that $(y_1, \alpha_1) \neq (y_2, \alpha_2) \Rightarrow H(y_1, \alpha_1) \neq H(y_2, \alpha_2)$. Let $(y_1, \alpha_1) \neq (y_2, \alpha_2)$. We need to consider two cases. First, let $\alpha_1 = \alpha_L$ and $\alpha_2 = \alpha_H$. Then,

$$\begin{aligned} H(y_1, \alpha_1) &= H(y_1, \alpha_L) = h(y_1) < \Delta \\ H(y_2, \alpha_2) &= H(y_2, \alpha_H) = h(y_2) + \widehat{r} > \Delta. \end{aligned} \tag{54}$$

Thus, $H(y_1, \alpha_1) \neq H(y_2, \alpha_2)$. Next, consider the case where $y_1 \neq y_2$ and $\alpha_1 = \alpha_2$. If $\alpha_1 = \alpha_2 = \alpha_L$, then $H(y_1, \alpha_1) = H(y_1, \alpha_L) = h(y_1)$, and $H(y_2, \alpha_2) = H(y_2, \alpha_L) = h(y_2)$. Since h is injective, $H(y_1, \alpha_1) = h(y_1) \neq h(y_2) = H(y_2, \alpha_2)$. Thus, in both cases $H(y_1, \alpha_1) \neq H(y_2, \alpha_2)$.

Claim 2: $H(y, \alpha)$ is surjective.

Here we have to show that $\forall z \in (0, \Delta) \cup (\widehat{r}, \widehat{r} + \Delta)$, $\exists (y, \alpha)$ such that $H(y, \alpha) = z$. Let $z \in (0, \Delta) \cup (\widehat{r}, \widehat{r} + \Delta)$. Then, either $z \in (0, \Delta)$ or $z \in (\widehat{r}, \widehat{r} + \Delta)$. If $z \in (0, \Delta)$, since h is surjective, there exists $y \in \mathbb{R}$ such that $h(y) = z$. Thus, $H(y, \alpha_L) = h(y) = z$. Next, suppose that $z \in (\widehat{r}, \widehat{r} + \Delta)$. Then, $\widehat{r} < z < \widehat{r} + \Delta$, thus, $0 < z - \widehat{r} < \Delta$. Again, since h is surjective, there exists $y \in \mathbb{R}$ such that $h(y) = z - \widehat{r}$. Thus, $H(y, \alpha_H) \equiv h(y) + \widehat{r} = z$. Thus, $H(y, \alpha)$ is surjective.

Claim 3: $H(y, \alpha)$ is continuous.

It suffices to show that the inverse image of every basis (open) set is open in the respective topology. Note that any open set in the range of H is of the form $(a, b) \cap \{(0, \Delta) \cup (\widehat{r}, \widehat{r} + \Delta)\}$, with $a, b \in \mathbb{R}$. First, consider any open set, B, such that $B \subset (0, \Delta)$. Then, $h^{-1}(B)$ is open in $\mathbb{R}$, since h is continuous. We have

$$\begin{aligned} & (y, \alpha) \in H^{-1}(B) \\ \Leftrightarrow \quad & H(y, \alpha) \in B \subset (0, \Delta) \\ \Leftrightarrow \quad & \alpha = L, H(y, \alpha) = h(y) \in B \\ \Leftrightarrow \quad & \alpha = \alpha_L, y \in h^{-1}(B) \\ \Leftrightarrow \quad & (y, \alpha) \in h^{-1}(B) \times \{\alpha_L\}. \end{aligned} \tag{55}$$

Thus, $H^{-1}(B) = h^{-1}(B) \times \{\alpha_L\}$, thus, whenever B is open, we have that $H^{-1}(B)$ is open in the product topology, since $h^{-1}(B)$ is open in $\mathbb{R}$, and $\{\alpha_L\}$ is open in $\{\alpha_L, \alpha_H\}$. Proceeding in the same fashion, we can show that the inverse image of B is open for any open set $B \subset (\widehat{r}, \widehat{r} + \Delta)$. The only remaining case involves a set $B = B_1 \cup B_2$, where $B_1 \subset (0, \Delta)$ and $B_2 \subset (\widehat{r}, \widehat{r} + \Delta)$. This easily follows from combining the above two cases.

Finally, we have the following.

Claim 4: $H^{-1}(y, \alpha)$ is continuous.

We must show that $H(B)$ is open for every basis (open) set B. Let $B = (y_1, y_2) \times \{\alpha\}_{\alpha \in \{\alpha_L, \alpha_H\}}$ be an open set in the product topology $\mathbb{R} \times \{\alpha_L, \alpha_H\}$. First, suppose that $\alpha = \alpha_L$; i.e., $B = (y_1, y_2) \times \{\alpha_L\}$. Then,

$$H(B) = H\left((y_1, y_2), \{\alpha_L\}\right) = h\left((y_1, y_2)\right). \tag{56}$$

Thus, $H(B)$ is open in $\mathbb{R}$, since h^{-1} is continuous and $h((y_1, y_2))$ is open. Similarly, if $\alpha = \alpha_H$; i.e., $B = (y_1, y_2) \times \{\alpha_H\}$, we have

$$H(B) = H\left((y_1, y_2) \times \{\alpha_H\}\right) = \{h(y) + \widehat{r} : y \in (y_1, y_2)\}. \tag{57}$$

Let $z = h(y) + \widehat{r}$ and $y \in (y_1, y_2)$. Then, $z - \widehat{r} = h(y) \in h((y_1, y_2))$. Since h^{-1} is continuous, $h((y_1, y_2))$ is open in $\mathbb{R}$. Thus, there exists an open interval (z_1, z_2) such that

$$z - \widehat{r} \in (z_1, z_2) \subset h((y_1, y_2)). \tag{58}$$

This implies that

$$z \in (z_1 + \widehat{r}, z_2 + \widehat{r}) \subset \{h(y) + \widehat{r} : y \in (y_1, y_2)\}. \tag{59}$$

Thus, $H(B) = \{h(y) + \widehat{r} : y \in (y_1, y_2)\}$ is open in $\mathbb{R}$.

We conclude that $H(y, \alpha)$ is a homeomorphism. ■

References

[1] Amador, M., and P. Weill (2007) "Learning from Prices: Information Aggregation and Central Bank Communication," Manuscript.

[2] Barro, R.J., and D.B. Gordon (1983) "A Positive Theory of Monetary Policy in a Natural Rate Model," Journal of Political Economy, Volume 9, Number 4, pp. 589-610.

[3] Berentsen A., and C. Monnet (2007) "Monetary Policy in a Channel System," Manuscript.

[4] Cooper, R. (1999) "*Coordination Games: Complementarities and Macroeconomics,*" Cambridge, Cambridge University Press.

[5] Keynes, J.M. (1936) *The general Theory of Employment, Interest, and Money*, London: Macmillan.

[6] King, R., Y.K. Lu, and E. Pasten (2008) "Managing Expectations," forthcoming in the Journal of Money, Credit, and Banking.

[7] Kiyotaki, N., and R. Wright (1989) "On Money as a Medium of Exchange," Journal of Political Economy, Volume 97, Number 4, pp. 927-954.

[8] Lagos, R., and R. Wright (2005) "A Unified Framework for Monetary Theory and Policy Analysis," Journal of Political Economy, Volume 113, Number 3, pp. 463-484.

[9] Lucas, R.E. (1972) "Expectations and the Neutrality of Money," Journal of Economic Theory, Volume 4, Number 2, pp. 103-124.

[10] Phelps, E.S. (1983) "The Trouble with Rational Expectations and the Problem of Inflation Stabilization," in R. Frydman and E.S. Phelps, eds., *Individual forecasting and aggregate outcomes.* New York: Cambridge University Press, pp. 31-40.

[11] Rajan, R. (2005) "Has Financial Development Made the World Riskier?" NBER Working Paper Number 11728.

[12] Rocheteau, G., and R. Wright (2005) "Money in Search equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," Econometrica, Volume 73, Number 1, pp.175-202.

[13] Weiss, L. (1980). "The Role for Active Monetary Policy in a Rational Expectations Model," The Journal of Political Economy 88(2), 221-233.